Automated design of coordinate system for long linear objects

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Abstract. In this paper we present a concise theory of geodetic design projections for area and linear objects on the Earth. The formulas given in the paper allow representing areas that not exceed 16^0 in longitude and latitude. A case study was carried out in order to show the advantages of our approach. It is possible to design a coordinate system for a linear object that is located within three 6^0 zones of the Gauss-Kruger projection or a small unique object with extra high accuracy. The methodology presented allows creating coordinate systems for long linear objects with minimal distortions and required accuracy of calculations.

1 Introduction

Modern measurement and calculation tools in geodesy, as well as new methods of data collection about objects located on the Earth surface and above it, provide new possibilities for the implementation of cartography databases improving rational use of natural resources. A great role should belong to technologies representing spatial information about objects on the surface of Earth and above it in the GIS. In order to get a reliable representation of information we need a mathematical description, which is suitable for using in the modern software. Coordinate systems that allow us to achieve a high accuracy in the description of spatial information should be used in geographic information technologies. From our point of view such coordinate systems should be simple in practical use.

Traditionally, most surveys used an assumed or local plane coordinate system. It does not accommodate splitting a large project into smaller independent projects. The approximation of the surface of the earth to a plane is valid for only a limited extent of an area. Beyond that, the corrections have to be applied to distances and angle to reflect the deviation of the curved earth from a plane. For small and isolated projects, such as local property surveys or construction surveys, an assumed plane coordinate system can be acceptable. However, for large engineering projects, such as a lengthy highway, this practice must be avoided. [1]

We have developed a methodology for cartographic projections designed to create a coordinate system. The theory is the most suitable for representation of the Earth surface areas and linear objects in GIS. In this paper we review some design features of coordinate systems for linear objects in automated systems. They are based on a new class of projections obtained by means of a linear composition of conical and cylindrical projections. The general principles of projections' design for GIS have been listed in the paper [2]. One can find requirements for modern coordinate systems applied in GIS there.

It is possible to create governmental, regional and local rectangular systems of coordinates that connected to each other. Such coordinate systems can be appreciated for different purposes of GIS.

The theory developed by us allows to solve following tasks:

- creation of a coordinate system for GIS that suitable for representation of information on the plane of geodetic projection;
- connection for different rectangular systems of coordinate used in different countries nowadays;
- chose an optimal system of coordinates for conditions that defined to solve the task.

It is important to note that the approach to development projections for linear objects is the same as for the representation of area object in special cases of most common geodetic projections (Gauss-Kruger cylindrical projection, Lambert conical projection, Russell stereographic projection). A number of design features dealing with the creation of projections that are the most suitable for linear objects from the point of view of minimum distortions are also described. The approach is founded on the theory of composite projection design [3, 4]. Special attention will be paid to the calculation of the coefficients of the composite projection.

2 General methodology of development of geodetic projections

We have developed a general theory presented in paper [3] with the aim to describe a certain class of conformal projections. Our methodology considers most common geodetic projections as special cases (Gauss-Kruger cylindrical projection, Lambert conical projection, Russell stereographic projection). We also consider possibility to generate new projections with minimal distortions possible and with any required parameters that a user can give.

2.1 The rectangular plane coordinates' calculation consequence in a chosen projection

In work [3] coordinates *x*, *y* are expressed by the following relations

$$x = x_0 + \sum_{j=1}^{n} c_j P_j$$
$$y = y_0 + \sum_{j=1}^{n} c_j Q_j \qquad \textbf{(1)}$$

In equation (1) x_0, y_0 are the coordinates of the initial point of the projection. We assume that x_0 and y_0 can be set equal to the length of the meridian arc and the ellipsoid parallel from the equator and the Greenwich meridian correspondently, or to any fixed numerical value depending on the current task. What concerns coefficients c_j defining the type of projection, one can find them in paper [2] for cylindrical projections to n=8 and ninth coefficient expresses following formula

$$c_9 = \frac{c_1}{362880}\cos^8 B_0 \cdot (1385 - 19028\,tg^2 B_0 + 18270\,tg^4 B_0 - 1636\,tg^6 B_0 + tg^8 B_0) \qquad \textbf{(2)}$$

Here B_0 – latitude of a central point in projection, the formula for c_1 one can find in paper [2] as well.

For conical projections it is possible to calculate c_j using following recurrent formula

$$c_j = \frac{c_1}{j!}(-1)^{(j-1)}(\sin B_0)^{(j-1)}\,(j = 1,2,3,...,n). \qquad \textbf{(3)}$$

The P_j and Q_j are calculated from harmonic multinomial equations that satisfy to Laplace equations

$$\begin{aligned} P_j &= P_{j-1}P_1 - Q_{j-1}Q_1 \\ Q_j &= P_{j-1}Q_1 + Q_{j-1}P_1 \end{aligned} \qquad \textbf{(4)}$$

where $P_1 = \Delta q = q - q_0$ (q – isometric latitude of current point and q_0 – isometric latitude of central point in projection), $Q_1 = \Delta L = L - L_0$ (L – longitude of current point and L_0 – longitude of central point in projection), $P_0 = 1$, $Q_0 = 0$ and q can be expressed from the following equation

$$q = \ln\sqrt{\left(\frac{1+\sin B}{1-\sin B}\right)\left(\frac{1-e\sin B}{1+e\sin B}\right)^e} \qquad \textbf{(5)}$$

In equation (5) are given next designations: B – latitude of current point, e – the first eccentricity of ellipsoid.

2.2 Formulas for isometric coordinates' calculation from rectangular coordinates

We can use the following expressions to calculate isometric coordinates on the surface of the Earth ellipsoid from plane rectangular coordinates

$$q = q_0 + \sum_{j=1}^{n} c_j' P_j'$$
$$L = L_0 + \sum_{j=1}^{n} c_j' Q_j' \qquad (6)$$

where

$$P'_j = P'_{j-1} P'_1 - Q'_{j-1} Q'_1$$
$$Q'_j = P'_{j-1} Q'_1 + Q'_{j-1} P'_1 \qquad (7)$$

In calculations with equation (7) we should assume $P'_0 = 1$, $Q'_0 = 0$, $P_1' = \Delta x = x - x_0$ (x – rectangular coordinate of current point, x_0 – rectangular coordinate of central point in plane of projection), $Q_1' = \Delta y = y - y_0$ (y – rectangular coordinate of current point, y_0 – rectangular coordinate of central point in plane of projection) .

The isometric latitude of a point with coordinates x_0, y_0 is calculated by the following expression

$$q_0 = \ln \sqrt{\left(\frac{1+\sin B_0}{1-\sin B_0}\right)\left(\frac{1-e\sin B_0}{1+e\sin B_0}\right)^e} \qquad (8)$$

Here B_0 – latitude of central point that calculates by formulas for iterations given in paper [3], e – the first eccentricity of ellipsoid.

In expression (6) coefficients c'_j are the coefficients of the inverted power series. They have been got up to n=9 and are presented in paper [5].

2.3 General principles of composite projection design

We have developed a methodology for a design of the composite projections that satisfy the Chebyshev-Grave criterion of the best projection.

A projection that is formed by a combination of cylindrical and conical projection is called composite geodetic projection. Coordinate equations for such projection are presented by the following expression

$$X = k_1 X_1 + k_2 X_2$$
$$Y = k_1 Y_1 + k_2 Y_2 \qquad (9)$$

where k_1 and k_2 are the coefficients of the cylindrical and the conical projections respectively while X_1, X_2, Y_1, Y_2 – coordinates of projections that formed composite projection.

It is also important to note that coefficients for cylindrical and conical projections satisfy following formula

$$k_1 + k_2 = 1 \qquad (10)$$

The equation (10) that expresses relation between conical and cylindrical coefficients serves to simplify their findings.

3 Case study

3.1 Data

We have chosen a linear object that is located in Europe and crosses Poland and Germany. It is shown in figure 1. The object consists of eight points. Each point has its own geographic coordinates *B, L* (*B* – latitude and *L* – longitude). In our case we have a set of coordinates that serve as data for the linear object. We need to have a datum, or basic parameters of ellipsoid, for implementation of our case study. We use the Krassowski ellipsoid with the following parameters: semi major axis a=6378245 m and flattening 1/f=298.3. The geographic coordinates of the points that belong to our linear object are presented in table 1.

Fig. 1. Example of linear object

Table 1. Geographic coordinates of linear object

Point	***B***	***L***
Warsaw	$52^0 16'$	$21^0 00'$
Kydz	$51^0 46'$	$19^0 27'$
Poznan	$52^0 24'$	$16^0 53'$
Berlin	$52^0 31'$	$13^0 31'$
Hanover	$52^0 24'$	$9^0 43'$
Bielefeld	$52^0 01'$	$8^0 31'$
Dortmund	$51^0 30'$	$7^0 28'$
Cologne	$50^0 57'$	$6^0 57'$

3.2 The Method

The projection development is based on the theory described in chapter 2 of this paper. The methodology considers some steps for the design of the best projection for linear objects. Let us consider the order of calculations in our example.

It is important to correctly to choose a central point for the coordinate system of our linear object. Usually, coordinates of a central point are calculated by the mean of the following formulas

$$B_0 = \frac{B_N + B_S}{2}$$
$$L_0 = \frac{L_E + L_W}{2} \quad \textbf{(11)}$$

where B_N and B_S are the latitudes of extreme points (to the North and to the South), L_E and L_W are longitudes of extreme points (to the East and to the West).

We then calculate the preliminary rectangular coordinates needed to find the equation of the regression line for our object. The coefficients of participation for cylindrical and conical projections will be dependent on the regression line. Coefficients of the regression line are calculated by the means of following expressions:

$$Det = N * [x^2] - [x]^2 \quad \textbf{(12)}$$

$$a = \frac{[x^2]*[y] - [x]*[x*y]}{Det} \quad \textbf{(13)}$$

$$b = \frac{N*[x*y] - [x]*[y]}{Det} \quad (1\textbf{4})$$

Here N – a number of points that involved in calculations, x, y – preliminary coordinates of current point in standard projection (cylindrical projection of Gauss-Kruger, conical projection of Lambert or stereographic projection Russell), [] – signification of sum.

A line of regression can be written following formula

$$y = b * x + a \quad \textbf{(15)}$$

And we can calculate coefficient for conical projection using next formula

$$K_{conic} = -\left|\frac{b^2}{1+b^2}\right| \quad or \quad K_{conic} = +\left|\frac{1+b^2}{b^2}\right| \quad \textbf{(16)}$$

Coefficient for cylindrical projection can be expressed via following relation

$$K_{cylindr} = 1 - K_{conic} \quad \textbf{(17)}$$

In equation (16) calculations of coefficient for conic projection k_{conic} should be executed according to the first expression if value B has a sign “minus” or according the second expression in case value B has a sign “plus”.

Next we are giving the values that have been calculated for our case study in table 2

Table 2. The results of calculations for regression line

Det	*a*	*b*	k_{conic}	$k_{cylindr}$
1,65342*10^11	-21239889,69	3,6655	1,074426784	-0,074426784

And the equation of regression for our linear object express following formula

y = 3,6655 x - 21239889,169

3.3 Results

Through the input of necessary parameters in software we have obtained rectangular coordinates on the plane in the best suitable projection. In our case study we have designed a new composite geodesic projection with coefficients of participation of k_1=1,0744 and
k_2=-0,0744 for conical and cylindrical projections accordingly. As a result we have got each point's coordinates for our linear object and the numerical characteristics for each point such as γ – the Meridian Convergence and *m* – the scale of distortion. In table 3 we have presented the results obtained by the program.

Table 3. The rectangular coordinates and numeric characteristics of the linear object in the best suitable projection

Point	*X, m*	*Y, m*	*m*	*γ, ⁰*
Warsaw	5816114,5580	478282,1980	0,999837269	5,505259852
Kydz	5751609,3421	377025,8642	0,999870160	4,291559314
Poznan	5811874,8404	197349,6625	1,000037343	2,275297419
Berlin	5821040,5534	-31682,9503	1,000099839	-0,366105446
Hanover	5816437,4552	-290260,6473	0,999995933	-3,347468647
Bielefeld	5779344,7156	-374944,8066	0,999884561	-4,290464612
Dortmund	5728006,1252	-451902,7058	0,999821942	-5,117063385
Cologne	5670436,7918	-493508,8999	0,999876587	-5,525793498

The maximum scale distortion is 1/5600 for the point called Dortmund. It is a good result considering the fact that our linear object is located within three 6^0 coordinate zones. It is important to note that the accuracy obtained in our example enough to solve majority of engineering tasks.

4 Conclusions and suggestions concerning further work

The methodology for the development of the most suitable coordinate system for linear objects with minimal distortions is based on the theory of composite projection design [4]. Coefficients of conical and cylindrical projections that are used in composition we can easy get from the line of regression. The regression line can be determined by means of simple formulas used in statistical analysis. We can also notice that positioning of isocols (counters of equal distortions) is dependent on the location of the regression line.

The general algorithm [3] allows calculation of linear values with an accuracy not less than 0.001 m and less than $0.001''$ for angular values. The formulas presented allow us to represent areas that do not exceed 16^0 in latitude and longitude. For some tasks where we do not need high accuracy requirements the size of the represented area can be more than 16^0 in latitude and longitude.

The advantage of the approach is shown in our example. It is also shown that the representation of long linear objects located in several coordinate zones (such as in zone Gauss-Kruger projection which is still used on the territory of post Soviet Union) is possible for the placement of such an object in one coordinate zone with accuracy not worth than for zone representation. The maximum scale distortion for our case study is the same as if it were represented in three 6^0 zones of projection Gauss-Kruger. Our approach allows representing linear objects located in several (n=1, 2, 3, 4) zones in one zone for the whole object with a required accuracy.

The theory can be implemented to support coordinate systems for large engineering objects like length highway projects and so on. It is also possible to use the theory for small unique objects required extra high accuracy.

Our further work is going to deal with implementation of methodology, which helps to find the best suitable projections for modern software. We are going to find some ways to automate our algorithm development for purposes of GIS with minimal user intervention.

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