Investigation of the channel capacity of seafloor maps with coloured depth intervals

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Abstract. What are the optimum number of contour levels in a seafloor map with coloured depth intervals? This question is answered in terms of information theory. Information theory computes the channel capacity of an information source as the difference between the map entropy and the degree of misinterpretations (equivocation). In order to get the parameters needed for the entropy and equivocation computations, an experiment was carried out. From a digital seafloor model a series of maps were constructed. The topography of the seafloor was visualized as coloured depth intervals, and twenty two subjects participated in a perception study. From the experiment the channel capacity of the seafloor maps was found to be seven or eight classes, dependent on whether a smoothed or the original entropy graph is selected. The cartographic relevance of the channel capacity is discussed.

Keywords: Perception, information theory, map design

1 Introduction

What are the optimum number of contour levels in a seafloor map with coloured height intervals? This question is similar to the question of optimum number of classes in choroplet maps. There are several scientific approaches to this question. For example, the use of the maps can be brought to focus, i.e., investigate communication problems at the pragmatic level. Another approach is to investigate the perception of the map signs without evaluating their meaning or use, i.e., investigate communication problems at the syntactic level. Figures 1 and 2 illustrate what types of maps which will be considered in the present study.

Rød [Rød04] showed that arguments for the use of unclassed choroplet maps in preference to classed choroplet maps, are supported from a semiotic perspective. Arguments about accuracy can also be used to advocate unclassed maps [Rød04].

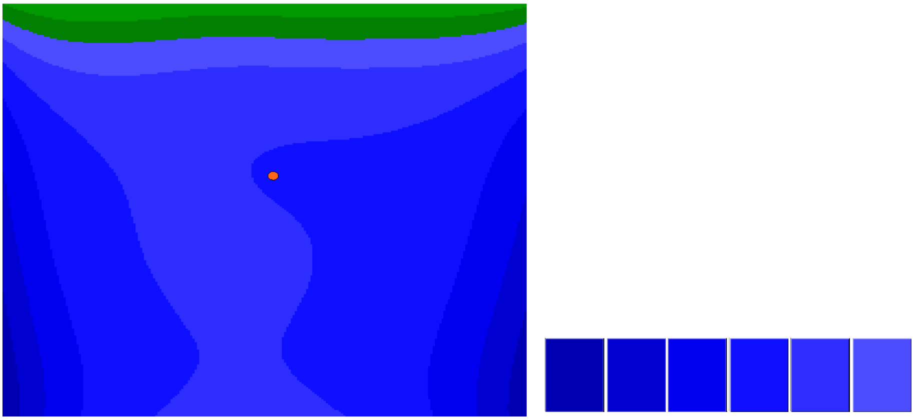

Fig. 1. Map example from the experiment. The sea floor model is divided into six depth classes which are associated a blue scale. The green colours represent the land area. The colours shown here can deviate slightly from the colours shown on the computer screens used in the experiment. The position of the boat is shown by a small circle in the middle of the window. When the map disappeared from the screen, the blue scale was shown and the subject had to point on the box that represented the colour of the depth class in the boats position.

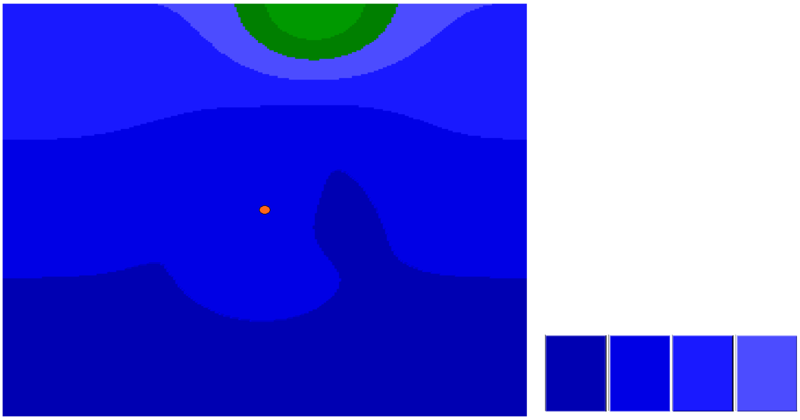

Fig. 2. Map example from the experiment. The sea floor model is divided into four depth classes which are associated a blue scale. The green colours represent the land area. The colours shown here can deviate slightly from the colours shown on the computer screens used in the experiment. The position of the boat is shown by a small circle in the middle of the window. When the map disappeared from the screen, the blue scale was shown and the subject had to point on the box that represented the colour of the depth class in the boats position.

Steinke and Lloyd [SL81] carried out a cognitive study to investigate how people compare choroplet maps. They found that blackness was the most important map attribute for judging the similarity of the maps used in the study, followed by correlation and map complexity. Mersy [Mer90] also takes a cognitive point of view and designed a human subjects study. In this study ten different map use tasks are applied. The two research papers cited, can be taken as representatives to demonstrate cognitive map studies which are dedicated to clarify how the map design affects the understanding and the use of the maps.

Pipkin [Pip75] takes an information theoretic point of view and proposes a method to quantify the amount of information the map reader has elicited from the map. An index I is computed as the difference between prior and posterior entropies, i.e., in Pipkins term the difference between prior knowledge and the knowledge after the map is perceived.

Miller [Mil56] investigates human perception from an information theoretic view, and he reviews several investigations where the information theoretic approach is used to measure the perceptual performance of subjects. In the experiments he reviews, the channel capacity for one dimensional stimulus was found to be seven plus minus two.

The approach of the present paper also is based on information theory and subjects testing. The goal of this research is to derive the channel capacity of seafloor maps with blue coloured depth intervals. This type of question is important in the design of maps to be used for safe boat navigation.

2 Methodology

Information theory [SW64] is not much used in cartography, but some case studies are carried out to demonstrate the application of information theory to cartographic generalization ([Bjø96], [B01], [LH02], [Bjø03], [BI05]).

The channel capacity of an information source is computed as

$$R = H(Y) - H(Y|X), \tag{1}$$

where $H(Y)$ is the entropy of the interpreted map and $H(Y|X)$ is the amount of confusion, i.e., the equivocation of the received message Y when information source X is used.

The entropy of the interpreted map is computed as

$$H(Y) = -\sum_{y \in Y} p(y) \log_2 p(y), \tag{2}$$

and the amount of confusion in the interpreted map is derived as

$$H(Y|X) = -\sum_{x \in X} p(x) \sum_{y \in Y} p(y|x) \log_2 p(y|x). \tag{3}$$

The computation of $H(Y)$ and $H(Y|X)$ requires that the transition probability $p(y|x)$ is known, i.e., we must know the probabilities that the different map colours are misinterpreted as well as correctly interpreted. Moreover, the probability p(x) must also be known, i.e., the probability that the different colours occur in the map.

The relation between $p(y)$ and $p(x)$ is derived from

$$p(y) = \sum_{x \in X} p(x) p(y|x). \tag{4}$$

Equation (3) can be written as

$$H(Y|X) = -\sum_{x \in X} p(x) H(Y|x),$$

where

$$H(Y|x) = -\sum_{y \in Y} p(y|x) \log_2 p(y|x). \tag{5}$$

I will term the quantity $H(Y|x)$ the local equivocation with respect to map symbol x, i.e., an expression for the equivocation introduced by a single map symbol.

Bjørke (1996) demonstrates two methods to compute the entropy of choroplet maps. The most intuitive method derives the probabilities for the different map colours from the size of the area they occupy. For example, if colour i covers an area of size a_i, the probability of i is derived as $p_i = a_i/A$, where A is the total area of the map. The drawback of this method is that the spatial correlation of the map colours is not considered. The other strategy computes the probability that the different colours are neighbours. To some degree the spatial correlation now is considered.

For the purpose of the present investigation, the simple method based on measuring the map area covered by the different map colours, will be selected. There are two reasons why I have decided to use the simple method. Firstly, in sea floor maps the depths can usually be regarded as highly correlated. Therefore, the depth intervals are not randomly distributed in the map. The method with differences are able to distinguish the correlated and the random case, but since this distinction is not important in sea floor maps, the simple method can be assumed to work well. The second reason is related to the derivation of the transition probabilities. This investigation is easier in the simple case. In the first case we need only the probability that one colour is interpreted as another. In the second case we must derive the probability that two colours are neighbours is interpreted as another neighbourhood.

In order to get an impression of what number of classes that corresponds to the channel capacity, an initial perception study was run [Sæh05]. This study shows that this number is six to eight classes. Based on the experience from the preliminary study, the number of colour scales to investigate were limited to scales from four and up to ten classes.

The colour scales were divided into equal step colour intervals from the dark blue to light blue. Based on the seven colour scales a series of maps with coloured depth intervals were constructed from a digital seafloor model. Since the visual variable colour lightness was used to portray the depth information, we obtained a logical relation between the selected visual variable and information variable.

Twenty two undergraduate students in geomatics participated in the investigation. A computer program was developed to generate the different maps. All the maps were generated from the same digital seafloor model. The subjects were shown each map in a time of period of one second. A red circle indicated the position of the boat. The map window was selected so that the circle appeared in the center of the window. The circle was never located close to the border of a depth interval, i.e., the possible confusion generated at the boundary between two colours were avoided. When the map disappeared, the colour scale used in the map was shown, and the subject was asked to identify the colour of the depth interval where the red circle had been located.

The seven colour scales selected, represent a total of 4+5+6+7+8+9+10=49 depth classes. Therefore, each subject was shown 49 different maps. In that way each subject did observe the whole range of the colour scales applied.

Table 1. Statistics for the subjects testing. The length, in terms of number of colours, for the different colour scales is shown together with the corresponding number of observations and their standard deviation σ. See the text for further explanation.

length of colour scale	number of observations	σ
4	88	0.38
5	110	0.56
6	132	0.69
7	154	0.88
8	176	0.94
9	198	1.42
10	220	1.27

3 Results

Statistics for the subject testing is shown in Table 1. The number of colours for the different colour scales is shown together with the corresponding number of observations and their standard deviation σ. In this case σ is computed relative the marked map colour, i.e., the difference between the screen colour and the observed colour of the depth interval at the position of the boat. Since σ is computed from all the observations of the considered colour scale, it represents the average accuracy for that scale. The unit of measurement for σ is colour index. The applied colour scales are designed so that they visually represent an ordering from light to dark, and the colours are numbered from 1 to n. Since the

Table 2. Transition probabilities for scales with four to seven colours

Scale with four colours				
sent	interpreted colour			
colour	1	2	3	4
1	**1.000**	0.000	0.000	0.000
2	0.045	**0.864**	0.091	0.000
3	0.000	0.409	**0.591**	0.000
4	0.000	0.000	0.045	**0.955**

Scale with five colours					
sent	interpreted colour				
colour	1	2	3	4	5
1	**1.000**	0.000	0.000	0.000	0.000
2	0.227	**0.636**	0.136	0.000	0.000
3	0.000	0.227	**0.545**	0.182	0.045
4	0.000	0.091	0.091	**0.727**	0.091
5	0.000	0.000	0.000	0.045	**0.955**

Scale with six colours						
sent	interpreted colour					
colour	1	2	3	4	5	6
1	**0.909**	0.091	0.000	0.000	0.000	0.000
2	0.091	**0.727**	0.091	0.091	0.000	0.000
3	0.000	0.136	**0.682**	0.182	0.000	0.000
4	0.045	0.091	0.364	**0.500**	0.000	0.000
5	0.000	0.000	0.000	0.318	**0.591**	0.091
6	0.000	0.000	0.000	0.045	0.182	**0.773**

Scale with seven colours							
sent	interpreted colour						
colour	1	2	3	4	5	6	7
1	**0.864**	0.136	0.000	0.000	0.000	0.000	0.000
2	0.136	**0.591**	0.182	0.091	0.000	0.000	0.000
3	0.000	0.091	**0.591**	0.273	0.045	0.000	0.000
4	0.000	0.091	0.273	**0.545**	0.091	0.000	0.000
5	0.045	0.045	0.182	0.273	**0.409**	0.000	0.045
6	0.000	0.000	0.045	0.000	0.273	**0.682**	0.000
7	0.000	0.000	0.000	0.000	0.045	0.182	**0.773**

Table 3. Transition probabilities for scales with eight to ten colours

Scale with eight colours								
sent	interpreted colour							
colour	1	2	3	4	5	6	7	8
1	**0.909**	0.091	0.000	0.000	0.000	0.000	0.000	0.000
2	0.045	**0.818**	0.136	0.000	0.000	0.000	0.000	0.000
3	0.000	0.227	**0.636**	0.091	0.045	0.000	0.000	0.000
4	0.000	0.091	0.182	**0.636**	0.045	0.045	0.000	0.000
5	0.000	0.091	0.091	0.227	**0.455**	0.136	0.000	0.000
6	0.045	0.000	0.000	0.000	0.318	**0.591**	0.045	0.000
7	0.000	0.000	0.091	0.000	0.000	0.227	**0.682**	0.000
8	0.000	0.000	0.000	0.000	0.000	0.136	0.182	**0.682**

Scale with nine colours									
sent	interpreted colour								
colour	1	2	3	4	5	6	7	8	9
1	**0.682**	0.182	0.091	0.000	0.045	0.000	0.000	0.000	0.000
2	0.091	**0.636**	0.182	0.045	0.045	0.000	0.000	0.000	0.000
3	0.000	0.045	**0.636**	0.091	0.227	0.000	0.000	0.000	0.000
4	0.000	0.045	0.091	**0.591**	0.136	0.091	0.045	0.000	0.000
5	0.000	0.000	0.318	0.500	**0.182**	0.000	0.000	0.000	0.000
6	0.045	0.000	0.045	0.091	0.227	**0.273**	0.182	0.091	0.045
7	0.000	0.000	0.136	0.273	0.091	0.136	**0.318**	0.045	0.000
8	0.000	0.045	0.045	0.000	0.000	0.045	0.182	**0.682**	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.091	0.318	**0.591**

Scale with ten colours										
sent	interpreted colour									
colour	1	2	3	4	5	6	7	8	9	10
1	**0.864**	0.091	0.045	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.136	**0.636**	0.182	0.045	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.227	**0.455**	0.227	0.045	0.045	0.000	0.000	0.000	0.000
4	0.000	0.000	0.091	**0.455**	0.273	0.091	0.091	0.000	0.000	0.000
5	0.000	0.000	0.136	0.091	**0.500**	0.136	0.091	0.045	0.000	0.000
6	0.000	0.045	0.136	0.182	0.273	**0.227**	0.136	0.000	0.000	0.000
7	0.000	0.000	0.091	0.136	0.136	0.045	**0.318**	0.182	0.091	0.000
8	0.000	0.045	0.000	0.000	0.000	0.000	0.045	**0.227**	0.682	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.045	0.091	**0.409**	0.455
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.227	0.318	**0.455**

visual distance from the one colour to the next is designed to be constant over the whole colour scale, it makes sense to compute σ from the colour index. The table shows that σ varies from 0.38 for the shortest scale and up to 1.42 for the longer scales.

From the experiment the transition probabilities are derived and shown in Tables 2 and 3. The intries of the tables are sent colour, i.e., the considered screen colour, and the interpreted colour. The colours on the diagonal of the tables are marked with bold face font. For example, for the scale with four colours Table 2 shows that when colour 2 is sent, the probability that this colour is interpreted as colour 1 is 0.045, colour 2 is 0.864, colour 3 is 0.091 and colour 4 is 0.000. The sum of the conditional probabilites is 0.045+0.864+0.091+0.000=1.000.

The local equivocation is derived from Equation 5 and plotted for each of the seven colour scales in Figure 3. In most of the cases the maximum degree of confusion occurs in the middle of the scale. Despite that the maximum point is moved to the one side for some of the colour scales, the main impression is that the maximum confusion occurs in the middle of the scale.

Figure 4 shows the entropy and equivocation computations. Here, the entropy $H(Y)$ is derived from Equation (2) and the equivocation $H(Y|X)$ is computed from Equation (3). The entropy is computed on the receiver side based on the assumption that all colours of a scale has equal probability in the map, i.e., the different $p(x)$ in Equation (4) are set $p(x) = 1/n$, where n is the number of colours in the colour scale considered. The assumption that $p(x) = 1/n$ is also introduced for the equivocation computation.

The R-value for the different colour scales is computed from Equation (1) and illustrated in Figure 5.

4 Discussions

The study is based on a limited number of subjects (22 subjects), and therefore outliers can affect the transition probabilities computed. Since it is hard to distinguish between outliers and natural variation, we have decided not to eliminate observations that does not fit the average observations well. Statistical analyzes of the derived transition probabilities are not worked out, since this is a topic for further study when more subjects are included in the study. The only data analysis applied, is represented by the models fitted to the entropy curves. In this way the effect of outliers are smoothed.

The equivocation computations presented in Figure 3, show that the maximum confusion is moved to the one side of the graph. The bias observed, can be explained on the basis of the colour intervals selected. Since there is a logarithmic relation between the energy in a light source and the perception of the colour, it may happen that the subjects had higher ability to differentiate the colours in the one end of the scale than in the other.

From the R-value computation, see Figure 5, the channel capacity is derived. The number of classes that corresponds to the channel capacity is 7 or 8, dependent on whether the smoothed or the observed R-value is used. Due to the

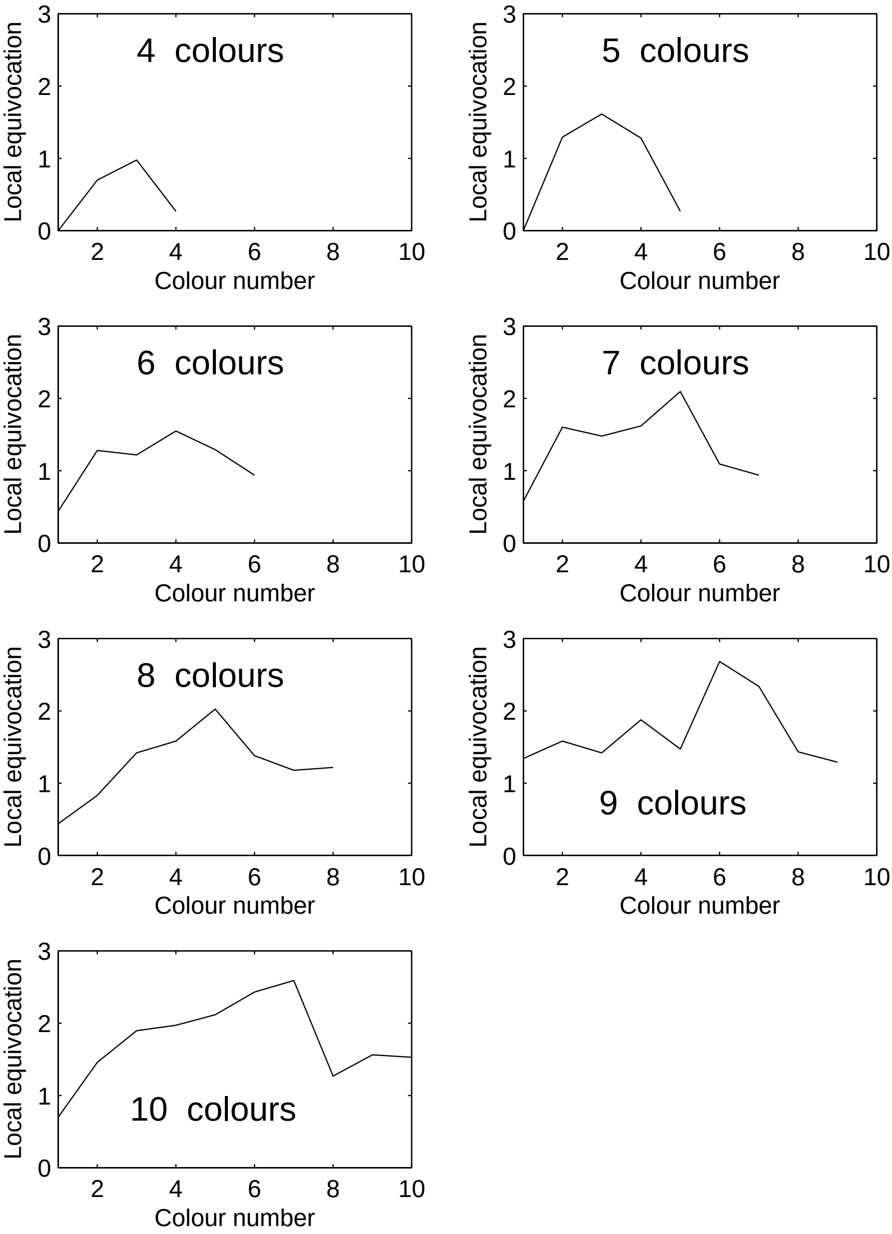

Fig. 3. Illustration of the local equivocation of the different colour scales in the subjects testing. The local equivocation is derived from Equation 5. It illustrates the degree of confusion introduced by the different colours.

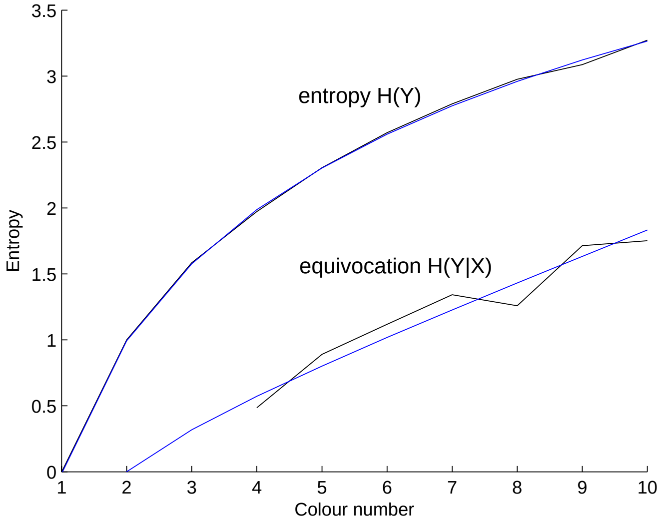

Fig. 4. Entropy computations based on the results from the subjects testing. The horizontal axis shows the number of colours in the different colour scales. For example, the scale with five blue colours gives the map entropy 2.2 bits, approximately. The black line corresponds to the observations, whereas the blue line is a smoothed version. The entropy $H(Y)$ is derived from Equation (2) and the equivocation $H(Y|X)$ is computed from Equation (3). The assumption that $p(x) = 1/n$ is introduced, see the text for further explanation.

limited number of subjects the statistical uncertainty of the material must be considered. The experiment is also affected by the selected colour scales. How sensitive the results are to changes in the colour scale, is not investigated. Since the main topic of the present study is to demonstrate a methodology to optimize map design, studies of the uncertainty and the sensitivity of the channel capacity is a topic for further experiments.

The crucial question is how the computed channel capacity can be interpreted. Is this just a number derived from a mathematical optimization, or has the channel capacity cartographic relevance? Arguments for the cartographic relevance will be given.

The parameters entered into the computation of the channel capacity are: (1) transition probabilities and (2) probabilities that the different colours occur in the map. Since these parameters are derived from studies of how map readers perceive maps and on analysis of the maps, they have cartographic relevance.

What about the mathematical formulation of the model? The map entropy can be interpreted as a variance, i.e., it can be taken as a measure of map complexity. The equivocation part of the model introduces changes of the variance due to the misinterpretations. Therefore, the model says that not all the com-

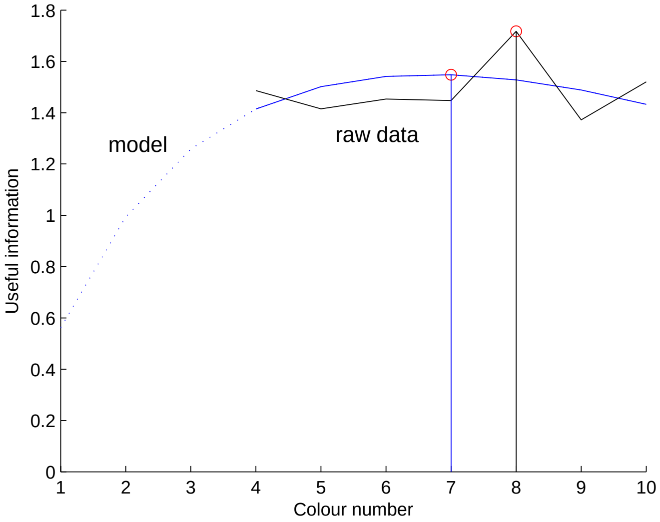

Fig. 5. Computation of the channel capacity based on the data derived from subjects testing. The horizontal axis shows the number of colours in the different colour scales. The R-value, i.e., the useful information, is computed from Equation (1). The black line represents the observed R-value, whereas the blue line is a smoothed version. The channel capacity corresponds to colour scales with 7 or 8 blue colours, dependent on which of the graphs are used.

plexity of the original map is transmitted to the map user. Since the entropy on the receiver side can be lower as well as higher than the entropy of the information source, due to the misinterpretations the map reader can underestimate or overestimate the complexity of the map. From this perspective an optimization based on information theory has cartographic relevance, i.e., the model computes the colour scale that transmits as much of the complexity of the source information as possible. The optimization considered, evaluates only the syntactic properties of the map colours, i.e., their separation. The semantic and pragmatic aspects of map communication is not considered in the present study.

5 Conclusions

When only the syntactic level of map perception is considered, it can be argued that the channel capacity has cartographic relevance. The number of seven or eight classes found in the present study, is derived from testing a small number (22) of subjects and more experiments are required to get a broad statistical material. Only blue colour scales are investigated. Probably, slightly different results will be obtained for other types of colour scales, for example the gray scale, the scale from yellow to red etc. This sensitivity of the channel capacity can be used to compare the cartographic efficiency of different colour scales.

The channel capacity of seven or eight classes is based on the assumption that the different intervals of each of the colour scales occupy areas of equal size. If that is not the case, this will affect the probabilities computed. Therefore, the channel capacity depends on the topography of the seafloor model. How this optimization can be used in map design, is a topic for further study.

If the concept of channel capacity is to be applied in geographical information systems for optimizing the visualization of continuous surfaces, the challenge is to derive the transition probabilities needed for the equivocation computation. This problem can be solved if some kind of training or learning system is implemented. In that way user feedback can be applied to compute the parameters needed for the optimization.

Acknowledgments

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