

# Application of Data Reduction Methods in Dynamic TIN Models to Topographic LIDAR Data

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**Abstract.** Comparisons of five data reduction methods associated with dynamic TIN models were conducted. All methods were applied to real world Light Detection and Ranging (LIDAR) topographic data. Data reduction is based on point selection by thresholding in dynamic Delaunay triangulation together with random point selection. The triangulation criteria used include Delaunay and hybrids of Delaunay and data dependent triangulation. The performance of the various reduction methods was evaluated by means of surface area, volume, RMS of vertical errors and maximum vertical errors. All methods were evaluated for five levels of reduction; 10%, 5%, 2.5%, 1% and 0.5% of full datasets.

## 1 Introduction

Although the processing capacity of desktop computers are continually increasing, the large data volumes generated by some data collection technologies such as Light Detection and Ranging (LIDAR), are posing a big challenge when it comes to processing, including spatial analyses and visualization. This makes a need for efficient data reduction methods. The objective of a data reduction method is to reduce the amount of data and at the same time preserve as much as possible of the useful information. A wide range of data reduction methods associated with polygon meshes are reported in the literature. However two main approaches for data reduction can be identified. The first main approach starts with a full model containing all data points and excludes the less significant points which are deemed redundant at the chosen level of decimation. The other main approach starts with a course mesh of some key points and successively adds data points which are deemed significant.

Examples of the first approach include the surface simplification method suggested by Schröder and Roßbach [1]. This method initially generates a high resolution TIN representing the given surface and then repeatedly reduces the number of points in this mesh. The criterion which is used to assess the significance of a point calculates a measure of the roughness of the terrain at that point. When the roughness value of a

point indicates that it can be removed from the mesh without affecting the overall representation significantly, the area around the removed point is retriangulated. The simplification algorithm proposed by Garland and Heckbert [2] is based on contraction of vertex pairs in a full model. Also Demaret et al [3] used an approach based on successive removal of points from a full model.

Examples of the second approach include adaptive triangular mesh (ATM) filtering proposed by Heller [4]. This approach can be considered as a conceptual equivalent to the Douglas-Peucker (DP) approach for lines. The triangulation starts with a few initial points. Depending on the vertical distance to the current triangular mesh, new points are added to the triangulation until no relevant points remain. Also Bottelier et al [5] use a combination of a "DP"-like algorithm and thresholding in a dynamic TIN model for near real time decimation of high density echo sounder data. This latter approach is by far more favourable from a processing demand point of view and the methods which are to be evaluated in this paper fall under this approach.

The performance of data reduction methods can be evaluated in terms of various criteria such as approximation errors and processing time. Garland and Heckbert [2] used error quadrics for the triangle planes joined at each vertex as a measure of approximation error. Demaret et al [3] used CPU time and maximum approximation error as evaluation criteria. Anderson et al [6] compared deviation in mean and standard deviation from full datasets to reduced datasets. Choice of criteria is highly application dependent. In visualization applications for example, one of the most important properties to preserve is the surface normal vectors Diebel et al [7]. In this study the class of polygonal meshes dealt with is confined to the Triangular Irregular Networks (TINs). The TIN model represents a surface as a set of contiguous non-overlapping irregularly shaped triangular facets. A TIN model can be regarded both as a terrain model as well as a data structure. It is a model because the space-filling triangular planar facets determine a value for the surface everywhere, but it also has a specific structure in which the points, triangles and topologies are stored [8]. The most common types of TIN models are Delaunay triangulations but also data dependent triangulations are well established.

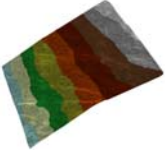
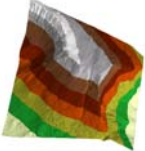

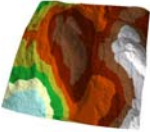
In order to discuss the advantages and disadvantages of TIN models it is natural to introduce grid models or digital elevation models (DEMs) as a reference. A DEM consists of a regular mesh of, usually square, cells. The TIN and DEM approaches are by far the most common types of terrain models. In general the TIN model is more complex with respect to algorithms and data structures and, dependent on implementation, may require more memory storage than a DEM. These are the main disadvantages of the TIN models. On the other hand the TIN model easily allow for variable resolution depending on topography. TIN can represent morphology directly and better than grid-based models, since they allow placing vertices at arbitrary positions, and embedding arbitrary poly-lines among their edges [9]. Also, using the sampled points directly and avoid the "gridding" process yield a more precise representation of a topographic surface as the introduction of errors associated with the "gridding" process are avoided. With this respect one can say that a TIN model is a model that fits the data and not vice versa. Moreover, many graphics systems are optimized for rendering triangles.

Comparison of data reduction methods in TIN models on real world topographic data has not gained too much attention in the literature. However, some research has been done e.g. Demaret et al [3]. The objective of the research reported in this paper is to quantify the performance of some data reduction methods on real world data. Five different data reduction methods; random subset, and two types of threshold in dynamic Delaunay triangulation, each combined with a final data dependent edge swap pass, have been applied to LIDAR topographic data. A common feature of all data reduction methods evaluated in this study is that they provide a subset of the full dataset by selection of points.

### 1.1 Description of Data

Analyses carried out in this study are based on data collected with airborne LIDAR. For a general description of the LIDAR technology, see for example Baltsavis [10] and the references therein. Technical specifications of the particular LIDAR system “Optech Airborne Laser Terrain Mapper (ALTM3100C)” used for data collection of this study can be obtained from [11]. The LIDAR data comprise point measurements; x-, y- and z-coordinates together with the intensity of the reflected signal. The background for this scan campaign is landslide hazard assessment. The area covered in this scan is along the fjord system of inner Storfjorden in Møre and Romsdal County, Norway. It covers nearly 600 square km, mostly steep hillsides from fjord to mountain. From this total area, 32 datasets each covering a square area of 1km by 1km, are selected as test datasets for this study. Data are pre-processed by the contractor and thus assumed to only contain bald earth measurement with no gross errors present. The point density of the test datasets ranges from 0.41 to 1.53 points per square meters with an average of 0.82 points per square metre. Some of the test areas have somewhat variable point density particularly where there is overlapping between two or more adjacent stripes. The difference of elevation from the highest to the lowest point within a single dataset varies from about 250 meters to about 1200 meters. In order to assess the performance of the data reduction methods on different topographies, the 32 test datasets were discretionary categorized into four categories. These categories are; “Hilly”, “Hillside”, “Ridge/Peak” and “Valley/Pit”. See Table 1 for a description of the categories and to see a sample of each category.

**Table 1.** Terrain categories. First row lists the category names, second row lists the category descriptions, third row provides an example of a representative dataset for the actual category.

“Hillside”	“Ridge/Peak”	“Valley/Pit”	“Hilly”
comprises planar slopes when the micro topography is disregarded	comprises concave terrain surfaces when the micro topography is disregarded	comprises convex terrain surfaces when the micro topography is disregarded	comprises broken and undulating terrain that does not fit into any of the other categories
			

## 2 Implementation and Calculations

### 2.1 Experiment

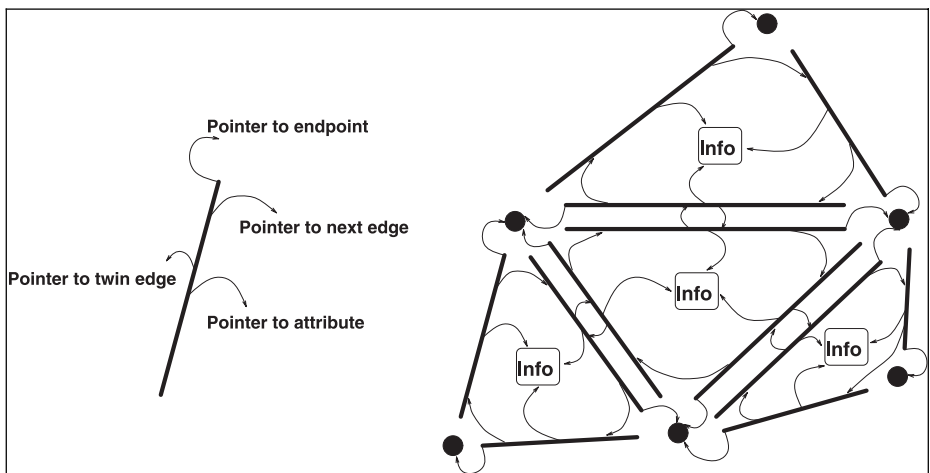
Five data reduction methods; RD, PD, PH, VD and VH (see Table 2 for details) were applied to 32 test datasets. Each dataset covers a square area of one by one kilometre with an average of 820.000 points per square kilometre. In order to evaluate the approximation errors of the various methods some criteria were chosen; surface area, volume, root mean square (RMS) of vertical residuals, mean of vertical residuals and maximum vertical residuals. Vertical residuals are calculated as the difference between the measured values of the control points and the corresponding interpolated values from the TIN surfaces. Since there was no accurate control measurements available the omitted points were used as control points. Surface area and volume calculations obtained from the reduced datasets are compared with those obtained from the corresponding full datasets. All methods were evaluated for five levels of reduction; 10%, 5%, 2.5%, 1% and 0.5% of full dataset.

**Table 2.** Specification of data reduction methods. A hybrid triangulation criterion means that the final Delaunay triangulation is modified by a data dependent edge swap pass. The embraced texts specify the data dependent swap criteria

Name	Point selection criterion	Triangulation criterion
RD	Random	Delaunay
PD	Perpendicular threshold	Delaunay
PH	Perpendicular threshold	Hybrid of Delaunay and data dep. (minimize vertical RMS)
VD	Vertical threshold	Delaunay
VH	Vertical threshold	Hybrid of Delaunay and data dep. (minimize max vertical error)

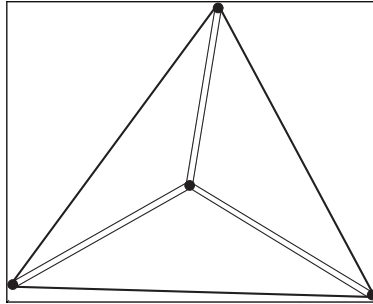
### 2.2 Description of TIN Data Structures and Algorithms

For the investigation reported in this paper a dynamic Delaunay triangulation algorithm was used as a basis. The dynamic algorithm starts with a course mesh of key points and allow for addition and removal of points during the triangulation process. See Midtbø [12] for a thorough description of data structures and algorithms for dynamic Delaunay triangulations. This basis algorithm was extended with functionality to accommodate the comparison study. Microsoft Visual Studio and C++ were used for the implementation. A twin edge data structure, proposed by Heller [4] makes up the backbone of the triangulation. All information about the triangles is implicitly stored in the edges and consequently no triangle table is required. However, to facilitate the triangle by triangle calculations needed for this study a triangle table was considered as a convenient extension. The topology of the network is maintained by pointers between edges (Figure 1).



**Figure 1.** Twin-edge data structure (figure copied from [12])

Initially, a triangulation of the corner points of a circumscribing rectangle of the dataset is chosen as approximation mesh. The height of these initial auxiliary vertices is arbitrary set to -300 meters, which results in selecting the points with largest elevation as the first inserted point in each of the two initial triangles. When a new point is inserted into the network the enclosing triangle of the current point is split into three new triangles and the data structure is updated accordingly, see Figure 2.



**Figure 2.** Insertion of a new point. The centre point is inserted and 6 new edges are added to the network (figure copied from [12])

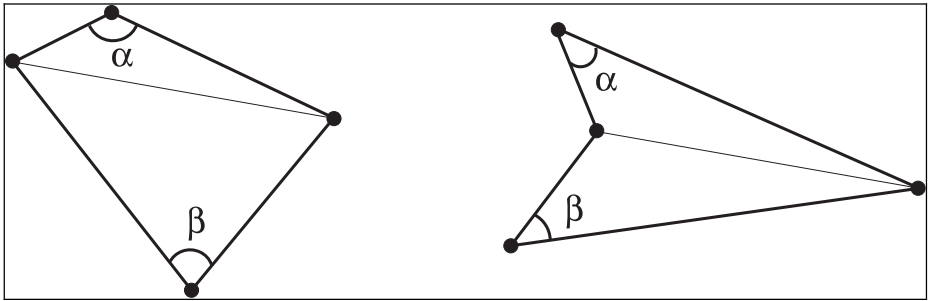
In order to maintain the Delaunay criterion when a new point is inserted, a recursive edge swap procedure is applied if the criterion is not met. Extension to data dependent edge swap is a minor adaptation of the existing Delaunay triangulation framework. The following pseudo code outlines the idea:

*Pseudocode for data dependent edge swap*

```

for each edge[i] element of Delaunay triangulation
    if ( isSwapable (edge [i]))
        if ( getCostCur (edge [i]) > getCostAlt (edge [i]))
            swapEdge (edge [i])
  
```

The `isSwapable` function is needed since not all quadrangles are convex and will therefore not generate consistent triangles when the diagonal is interchanged, match Figure 3.

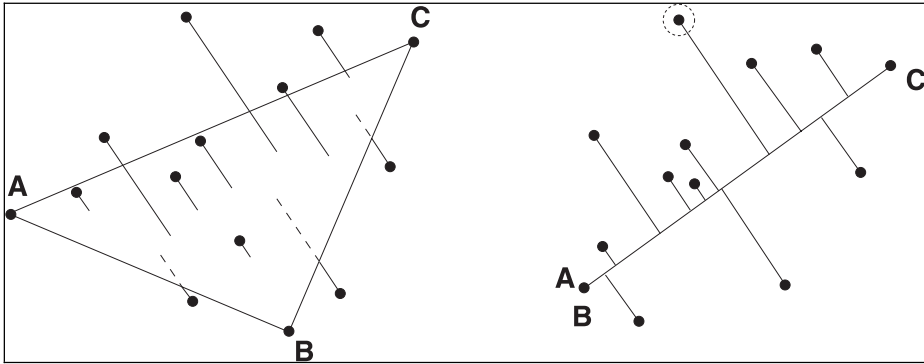


**Figure 3.** Quadrangles. The left quadrangle is convex and the diagonal is swappable. The right quadrangle is concave and the diagonal is not swappable (figure copied from [12])

The two functions `getCostCur` and `getCostAlt` calculate the costs associated with the two alternative configurations of the diagonal. The cost functions in case of method VH is maximum vertical residual. This will preserve the most useful property of the vertical distance method; that is to control the maximum vertical residual. The cost function in case of method PH is RMS. The `swapEdge` function performs the interchange of diagonal and updates the data structures.

### 2.3 Selection of Data Point

For the random selection criteria each data point is included in the TIN model with a probability  $p$ . For the vertical distance criterion, the point with maximum vertical distance from existing TIN surface is inserted into the triangulation, given that the distance is larger than some specified threshold (Figure 4). The procedure for the perpendicular criterion is identical to the vertical distance criterion, except that the vertical distance from the existing triangle plane is replaced by the perpendicular distance. Selecting the point with the largest perpendicular distance to the existing triangle plane corresponds to selecting the point having the most influence on volume for the splitting of this triangle. The point with the largest perpendicular distance forms the apex of an imaginary tetrahedron with the existing triangle plane as the base. Thus, this point is the one with the largest impact on volume. Both the vertical and the perpendicular distance criteria are combined with a final edge swap pass.



**Figure 4.** Point selection. The most distant point from the triangle plane will be inserted into the triangulation given that this distance is above some specified threshold (figure copied from [12])

## 2.4 Evaluation Criteria

In order to evaluate the approximation errors of the various methods some criteria were chosen; surface area, volume, root mean square (RMS) of vertical residuals<sup>1</sup>, mean of vertical residuals and maximum vertical residuals. Surface area calculations and volume calculations obtained from the reduced datasets are compared with those obtained from the corresponding full datasets. Vertical residuals are calculated as the difference between the measured values of the control points and the corresponding interpolated values from the TIN surfaces (3). The omitted data points are used as control points. To avoid boundary effects, such as removing points which belong to the convex hull of the point set, a considerable larger area than the “clipping window” (i.e. the square area of 1 by 1km) is subject to data reduction. This buffer zone is 200 metres. The effect of data reduction on the selected criteria is then evaluated only for the area inside the clipping window. The vast majority of all triangles are either completely outside or completely inside the square clipping window. These two groups are pointed out by integer tests concerning coordinate extent since the clipping windows are axis aligned. The remaining triangles can possibly intersect with the clipping window and if they do intersect, the output can in principle range from triangles to heptagons (seven-sided polygons) depending on the actual configuration of the input triangle and the clipping window. The clipping of this group of triangles are handled by a “Sutherland-Hodgman” polygon clipping algorithm (see for example [13]). Nevertheless all triangles clipped by the square window will become convex polygons. The convex property makes it straight forward to split polygons into triangles and the same framework for calculations of evaluation parameters can be applied to all triangles. The TIN model is used for calculation of all evaluation criteria. Surface area of each individual triangle is calculated by (1) and then summarized for

<sup>1</sup> In this paper both *residual* and *error* refer to the same i.e. deviation from full model.

all triangles to get the total surface area within the clipping window. The volume of a single tri-prism is calculated by (2) and the total volume is obtained by summarizing the volume of each individual tri-prism.

$$SA(\Delta) = \frac{1}{2} |\bar{a} \times \bar{b}|, \text{ where } \bar{a} \text{ and } \bar{b} \text{ are vectors that span the triangle facet.} \quad (1)$$

$$V(\Delta) = A(\Delta) \frac{(z_1 + z_2 + z_3)}{3}, \text{ where } A(\Delta) \text{ denotes the horizontal triangle area.} \quad (2)$$

$$z(x, y) = \frac{-ax - by - d}{c}, \text{ where a, b, c and d are calculated from the enclosing triangle.} \quad (3)$$

### 3 Results and Discussion

A large number of circumstances will affect the construction of a TIN model and consequently the derived parameters. Examples of factors influencing the results include; selection of initial points for triangulation, handling of degenerated cases, order of point insertion etc. Limited testing shows that some of these factors are inferior to others. However this problem will not be addressed in this study. In this study all factors not related to the test datasets it selves are hold constant. Selection of evaluation criteria is highly application dependent. This study has assessment of slide hazard in mind; volume calculations are of course essential to slide assessment and the ratio of surface area to projection area can serve as a simple measure of terrain roughness. The vertical residuals provide a more general measure of approximation quality. A more advanced interpolation technique such as Kriging would probably lead to better results with respect to interpolated height values but the intention is to give an idea of how well the TIN surface itself approximates the topographic surface.

Figure 5 shows the average difference in surface area together with the associated standard deviations. The hypotheses of interest in connection with surface area can be stated as follows:

$H_0$ : All methods have the same means within each level of reduction

$H_1$ : At least one of the methods has, for at least one level of reduction, a different mean than the other methods

In order to test these hypotheses, they are split into hypotheses regarding pair of methods. Thus the testing can be carried out by means of a number of paired Student's t-tests (t-tests). Two conditions are required for a paired t-test; 1) Each pair of observations is statistically independent from the other pair of observations 2) The differences between each pair of observations are approximately normally distributed. The first condition is assumed to be fulfilled since there is no overlap between the 32

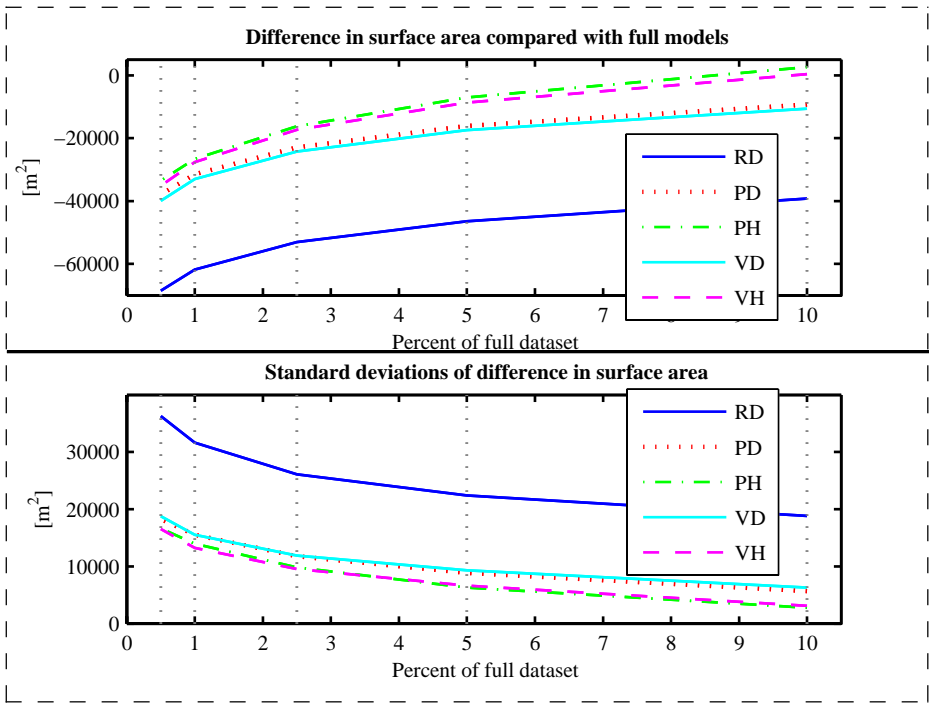
test datasets. According to histogram plots of the pair differences there are no indications of severe violations of the latter condition. These paired t-tests will involve all possible pair combinations of the five methods, i.e. first RD is tested against PD (for each level of reduction) than RD is tested against PH and so forth. The sub hypotheses are as follows:

$H_0: \mu_i = \mu_j$  (method  $i$  has the same mean as method  $j$  for reduction level  $k$ )

$H_1: \mu_i > \mu_j$  or  $\mu_i < \mu_j$  (method  $i$  has a larger/smaller mean than method  $j$  for reduction level  $k$ )

where  $i \in \{1..5\}, j \in \{1..5\}, k \in \{1..5\}, i \neq j$

The calculated difference value of the actual pair determines whether it should be a “less than” or a “greater than” test, thus the  $H_1$  hypotheses are one sided. Testing of the two latter hypotheses is separately conducted for each level of reduction, i.e. for 10%, 5%, 2.5%, 1% and 0.5% of full dataset. The testing is carried out with a significance level of 5% and 31 degrees of freedom. All five methods have significantly different mean values for every level of reduction. The mutual order of mean values are  $PH > VH > PD > VD > RD$  as suggested by Figure 5. Moreover, there is also an evident trend common to all methods; less data gives lower surface area. This is expected since a decrease in number of surface points will cause some smoothing of the finer topography, as well as a reduction in the random noise which also makes a significant contribution to surface area.

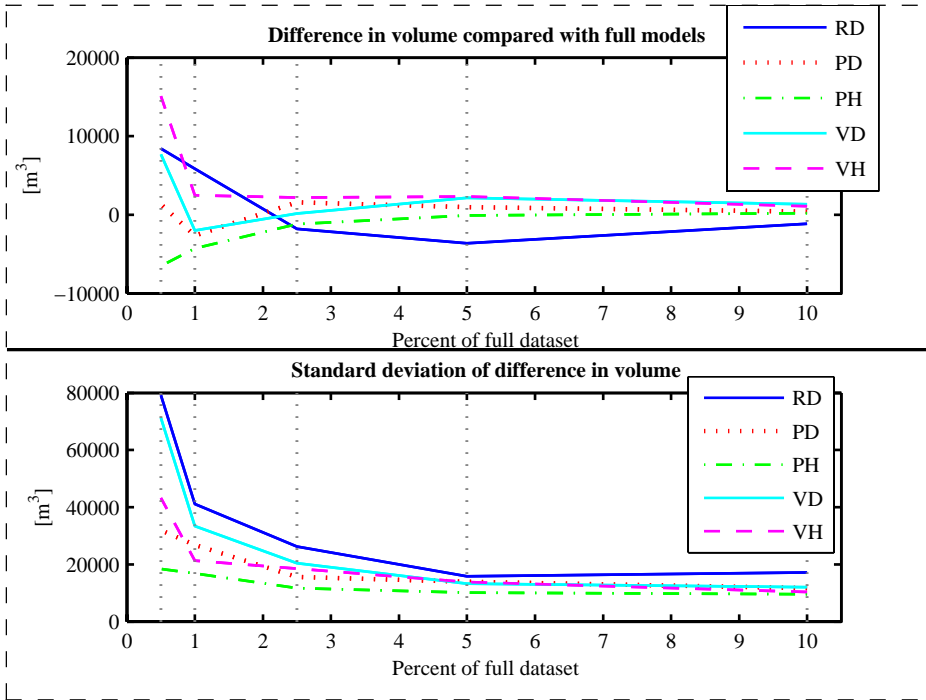


**Figure 5.** Surface area. Upper plot: average difference in surface area for the 32 test datasets. Lower plot: standard deviation of difference in surface area for the 32 test datasets. Vertical dotted lines indicate reduction levels applied for comparisons

Figure 6 shows the average difference in volume together with the associated standard deviations. Residual plots for the volume residuals indicate that they are closely normally distributed which makes the t-test applicable. The hypotheses to be tested are as follows:

- $H_0: \mu_i = 0$  (volume calculations associated with method  $i$  give unbiased estimates)
- $H_1: \mu_i \neq 0$  (volume calculations associated with method  $i$  give biased estimates)

Student’s t-tests with significance level of 5% and 31 degrees of freedom are used for testing. Each level of reduction is tested separately. None of the  $H_0$  hypotheses could be rejected at any level of reduction, which means that the volume calculations associated with all five data reduction methods are unbiased for all levels of reduction.



**Figure 6.** Volume. Upper plot: average volume difference compared with full models for the 32 test datasets. Lower plot: standard deviation of volume difference. Vertical dotted lines indicate reduction levels applied for comparisons

Figure 7 shows the vertical RMS values associated with the five methods. The RMS is tested pair by pair (as for the surface area) by means of a number of Fisher-tests (F-tests). In order to apply the F-tests, the RMS values are squared and thus assumed to be Chi-square distributed as implied by the F-test. The hypotheses are as follows:

$H_0$ :  $\sigma_i = \sigma_j$  (method  $i$  has the same RMS as method  $j$  for reduction level  $k$ )

$H_1$ :  $\sigma_i > \sigma_j$  or  $\sigma_i < \sigma_j$  (method  $i$  has a larger/smaller RMS than method  $j$  for reduction level  $k$ )  
 where  $i \in \{1..5\}, j \in \{1..5\}, k \in \{1..5\}, i \neq j$

The number of degrees of freedom is virtually unlimited if one consider all 32 datasets as a single area. The total number of data point processed is about  $2.6 \cdot 10^7$  where 0.5% to 10% are used for construction of the TIN models. The remaining 90% to 99.5% are used as control points and thus corresponds to degrees of freedom. This is a somewhat pragmatic adaptation since the different datasets do not contain the same number of points. Due to the large number of degrees of freedom all RMS values are significantly different at any reasonable significance level. For the reduction levels 0.5%, 1% and 2.5% the mutual order of the RMS values are  $RD > VD > VH > PD > PH$  where " $>$ " means significantly greater. However, for reduction

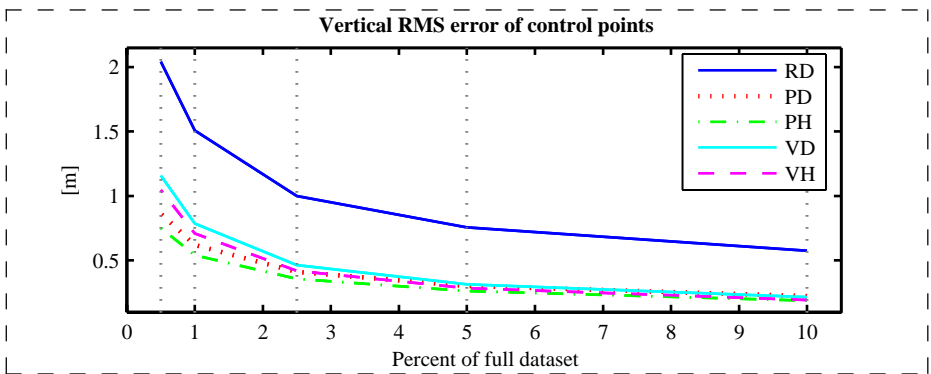
levels 5% and 10% this order is not valid. Still, RD has the highest RMS and PH has the lowest RMS but the order of the other methods is changed. Table 3 and Table 4 give comparisons of RMS for PD versus PH and for VD versus VH respectively.

**Table 2.** Vertical root mean square error of the control points for method PD and PH

Percent of full dataset	10%	5%	2.5%	1%	0.5%
PD RMS [m]	0.222	0.303	0.411	0.621	0.864
PH RMS [m]	0.189	0.262	0.357	0.540	0.752
Percent reduction of RMS	15.0	13.6	12.9	12.9	12.9

**Table 3.** Vertical root mean square error of the control points for method VD and VH

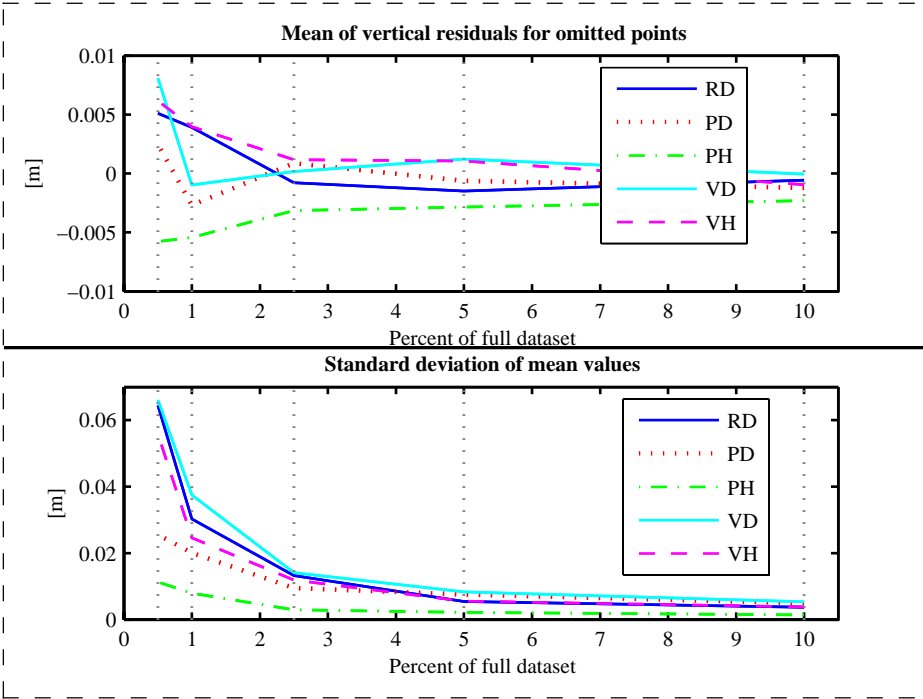
Percent of full dataset	10%	5%	2.5%	1%	0.5%
VD RMS [m]	0.216	0.314	0.463	0.789	1.158
VH RMS [m]	0.194	0.284	0.419	0.710	1.047
Percent reduction of RMS	10.1	9.5	9.7	10.0	9.6



**Figure 7.** Vertical root mean square residuals of the control points. Residual values are calculated as interpolated value from the TIN surface minus measured elevation. Vertical dotted lines indicate reduction levels applied for comparisons

Similar hypotheses and testing procedures as for volume were also applied to the mean of the vertical residuals. Residuals in this context refer to the deviation of elevation values of control points from those interpolated from the TIN surface. Hypotheses testing indicate that the mean of residuals are unbiased for all methods except for method PH. It was not expected that PH should be biased since the volume estimates based on PH are unbiased. This situation might be related to the variable

TIN resolution. Figure 8 shows mean values of the vertical residuals together with the associated standard deviations.



**Figure 8.** Mean of vertical residuals. Upper plot: mean values for the vertical residuals of control points. Lower plot: standard deviations of the mean values for the 32 test datasets. Vertical dotted lines indicate reduction levels applied for comparisons.

Figure 9 shows the average maximum vertical residuals together with the associated standard deviations. As for RMS, the max vertical residuals are squared in order to apply F-testing. Furthermore, the same pair wise testing scheme as for surface area is used. The hypotheses of interests are as follows:

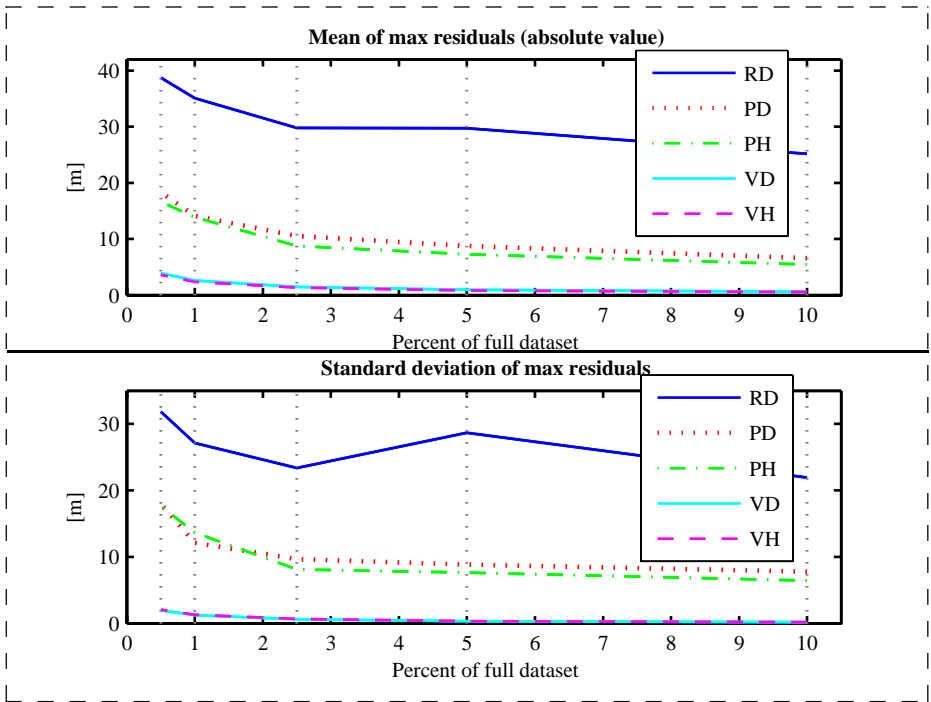
$H_0: \sigma_{\max_i} = \sigma_{\max_j}$  (method  $i$  has the same mean of max residuals as method  $j$  for reduction level  $k$ )

$H_1: \sigma_{\max_i} > \sigma_{\max_j}$  or  $\sigma_{\max_i} < \sigma_{\max_j}$  (method  $i$  has a larger > or smaller mean of max residuals than method  $j$  for reduction level  $k$ )

where  $i \in \{1..5\}, j \in \{1..5\}, k \in \{1..5\}, i \neq j$

Again, the calculated mean of maximum residuals of the actual pair determines whether it should be a “less than” or a “greater than” test, thus the  $H_1$  hypotheses are one sided. A significance level of 5% and 31 degrees of freedom are used for testing.

Testing shows that RD has a significant larger mean of maximum residuals than the other methods. Moreover, PD and PH show no significant difference in mean of max values. The same applies to VD and VH. Method PD and PH have a significant larger mean of maximum residuals than VD and VH. The results from hypotheses testing regarding mean of maximum residuals can be summarized as follows;  $RD > (PD \text{ and } PH) > (VD \text{ and } VH)$ , where “>” means significantly greater.



**Figure 9.** Max vertical residuals. Upper plot: means of the absolute value of the maximum vertical residual for the 32 test datasets. Lower plot: standard deviations of the maximum residuals. Vertical dotted lines indicate reduction levels applied for comparisons

The calculated evaluation parameters are dependent on the type of terrain and the properties of the actual datasets. However, no indications of significant change in *relative* performance (i.e. ranking of methods) of the five data reduction methods were found when applied to different types of terrain. However, this might be due to a poor categorization of terrain together with a small number of test datasets.

Although the Delaunay criterion produces a “good quality” triangulation in terms of its triangles' aspect ratios, it does not necessarily produce the triangulation which is the best approximation to a given height field for two reasons; 1) sliver triangles are necessary to give a good approximation to some surfaces and 2) the swapping of edges which the Delaunay criterion invokes can cause artificial break lines where

none exist in the original terrain [14]. A data dependent edge swap pass was performed for each of the two adaptive Delaunay triangulations. The processing cost associated with these edge swap passes is about 25% extra compared to the pure Delaunay triangulation. The gain of these data dependent edge swap passes is about 10-15 percent reduction of RMS error as well as better approximation of surface area. In accordance with the results in this paper, both Garland and Heckbert [15] and Rippa [16] report best results with a hybrid Delaunay – data dependent triangulation.

An ideal data reduction method should lead to unbiased results with low approximation errors as well as being cheap in terms of processing. Although the random subset method has some desirable properties, such as being unbiased for the parameters evaluated (except for surface area), the overall performance of this method is not adequate due to large variations in parameter estimates. This was expected, and the random method was introduced only to serve as a reference for the naive approach. The PH method has lower RMS than the PD method; this applies to all 32 test datasets as well as all 5 reduction levels. Similarly, the VH method has lower RMS than the VD method for all test datasets and for all levels of reduction. Statistical testing of PD RMS versus PH RMS is somewhat unnecessary since the data dependent edge swap is mostly a deterministic operation which solely aims to reduce the RMS. However there is a stochastic element involved since this is a greedy approach; during the edge swap process the same triangles can theoretically be subject to swapping several times and there is no guarantee that the optimal solution is found. The situation of VD and VH is slightly different; although there is a deterministic operation to swap diagonals to minimize the largest residuals; this is not directly related to the RMS which is to be evaluated.

Method PH has the smallest RMS and it also produces the largest surface area and lead to an unbiased volume estimate with the lowest standard deviation. Nevertheless, this method, as well as PD and RD, has a major drawback; it provides no control of the upper vertical approximation error. If this is considered as an important property only method VD and VH remain, where method VH provides the best overall approximation quality. However, it might be advantages to combine the vertical and perpendicular point selection criteria to get the best from both methods; slightly lower RMS and better approximation of surface area together with the ability to restrict the maximum vertical approximation errors. Some limitations of the applied methods should be mentioned; many conventional Geographical Information Systems (GIS) do not allow for dynamic triangulation and consequently these methods are not applicable for data reduction. Moreover the methods described in this paper is vulnerable to gross errors as they tend to give preference to gross errors if such are present in the data. Also, using a wider range of evaluation parameters including slope and curvature would have been beneficial.

## 4 Summary and Conclusions

Comparisons of five data reduction methods associated with dynamic TIN models were conducted. All methods were applied to the same test datasets of LIDAR topographic data. The performance of the various reduction methods were evaluated by means of surface area, volume, RMS of vertical errors and maximum vertical errors. Although the test datasets have somewhat variable properties with respect to topography and point density one can point out some general guidelines. Most of the graphs for the test parameters seem to have an evident deflection at 2.5% of full dataset. This gives an indication of a suitable upper level of reduction. No indications of significant change in *relative* performance of the five data reduction methods were found when applied to different types of terrain. However this might be due to a poor categorization of terrain together with a small number of test datasets. Depending on importance of evaluation criteria, method VH is considered as the most favourable. Method VH employs point selection by vertical threshold in dynamic Delaunay triangulation combined with a final data dependent edge swap pass to reduce the maximum vertical approximation errors. The final data dependent edge swap pass requires about 25% extra processing time with the reduction levels and implementation used for this study. The gain is a reduction of about 10-15% in RMS together with better approximation of surface area.

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